

Progression in Calculations

Addition

Key Vocabulary:

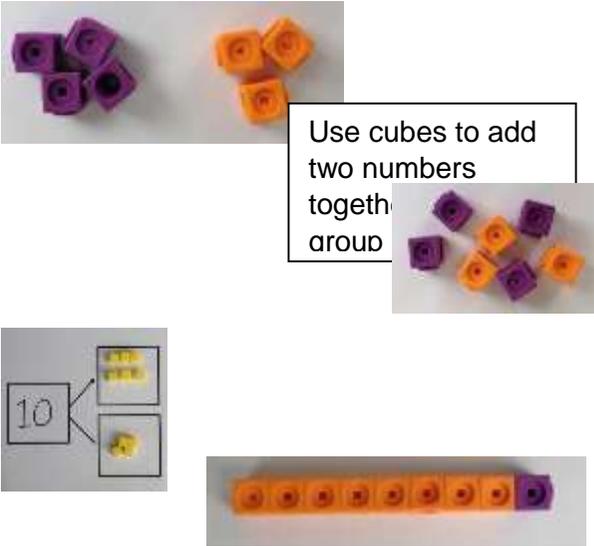
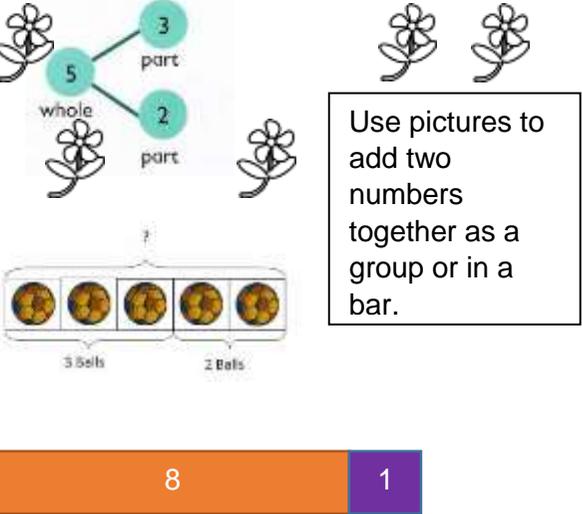
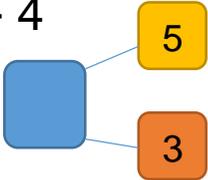
EYFS & Year 1: add, more, plus, and make altogether, total, equal to, equals, double, most, count on, number line.

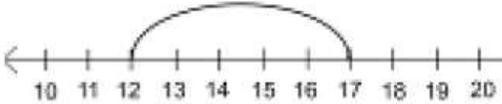
Year 2: As above plus: sum, tens, ones, partition, addition, column, tens boundary.

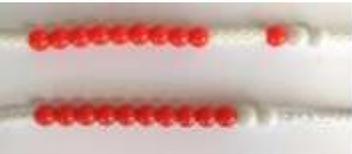
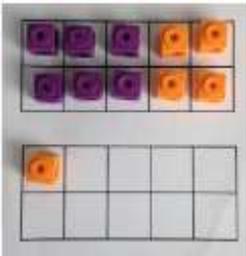
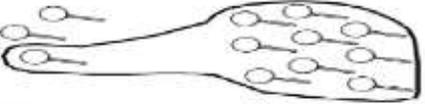
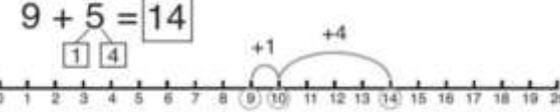
Year 3: As above plus: hundreds boundary, increase, vertical, "carry", expanded, compact, hundreds, digits.

Year 4: As above plus: thousands, inverse, decimal point, tenths, hundredths

Year 5 and 6: As above plus: decimal places, thousandths.

Taught at:	Objective and Strategies	Concrete	Pictorial	Abstract
EYFS and Year 1	Combining two parts to make a whole: part-whole model	 <p>Use cubes to add two numbers together</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-whole diagram as shown above to move into the abstract.</p>

<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
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<p>Regrouping to make 10.</p>	 <p>$6 + 5 = 11$</p>  <p>Start with the bigger number and use the smaller number to make 10.</p>	 <p>$3 + 9 =$</p> <p>Use pictures or a number line.</p> <p>Regroup or partition the smaller number to make 10.</p> <p>$9 + 5 = 14$</p> 	<p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
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Additional Resources

NRICH: [Incey Wincey Spider](#)

NRICH: [Number Lines](#)

NRICH: [Getting the Balance](#)

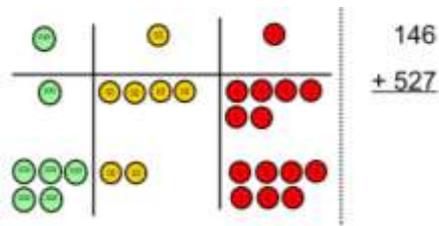
NRICH: [Ladybirds in the Garden](#)

Videos: www.tes.co.uk/teaching-resource/Number-bonds-to-ten-6413321

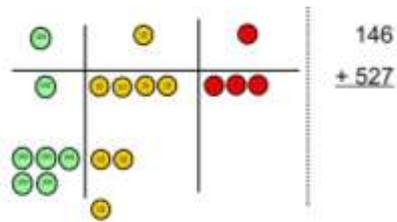
Years
3 to 6

Column
method-
regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

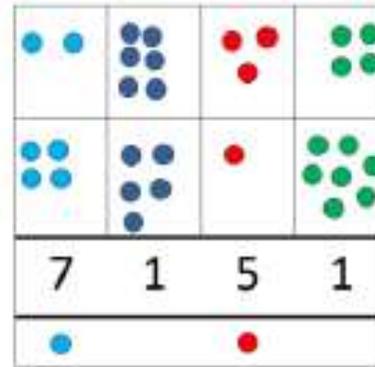


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 \\ 40 \\ 60 \\ \hline 120 \\ + 536 \\ \hline 656 \end{array} = 73$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Additional Resources

NRICH: [Super Shapes](#)

NRICH: [Roll These Dice](#)

NRICH: [Strike it Out](#)

NRICH: [Which Is Quicker?](#)

NRICH: [Reach 100](#)

NRICH: [Consecutive Numbers](#)

NRICH: Sea Level

Videos: www.teachertube.com/video/24325

Subtraction

Key Vocabulary:

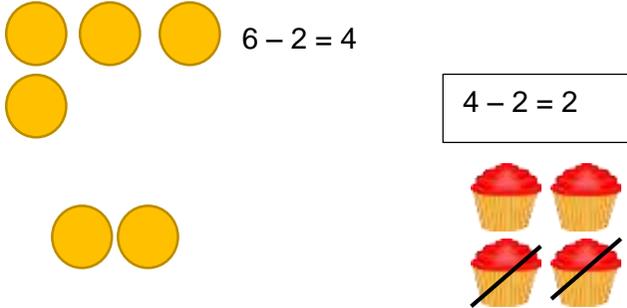
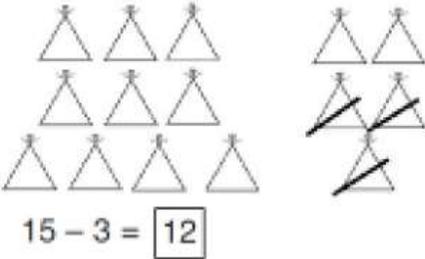
EYFS & Year 1: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/ less than, most, least, count back, how many left, how much less is __?, difference, count on, tens, ones.

Year 2: As above plus: strategy, partition.

Year 3: As above plus: exchange, decrease, hundreds, value, digit

Year 4: As above plus: inverse, tenths, hundredths, decimal point, decimal.

Year 5 and 6: As above plus: decimal places, thousandths.

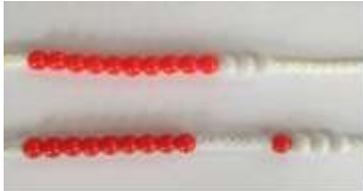
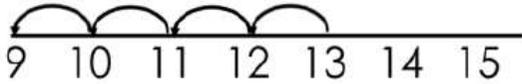
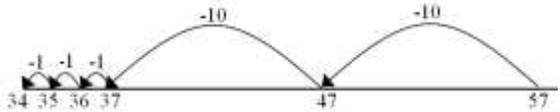
Taught at	Objective and Strategies	Concrete	Pictorial	Abstract
EYFS and Year 1	Taking away ones	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 2 = 4$</p> <p>$4 - 2 = 2$</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p>	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>

Additional Resources

NRICH: [Maths Story Time](#)

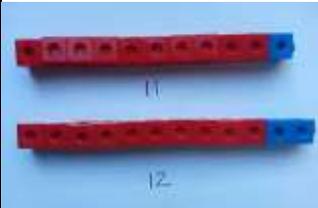
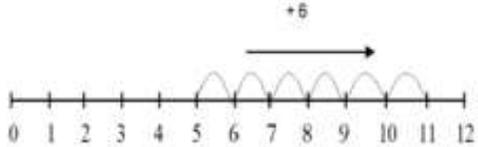
NRICH: [Using Incey Wincey Spider](#)

NRICH: [Using Golden Beans for Counting](#)

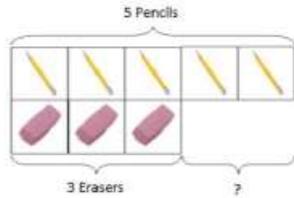
<p>Year 1 and Year 2</p>	<p>Counting back</p>	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>13 - 4</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>
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Additional Resources
 NRICH: [Super Shapes](#)
 NRICH: [Roll These Dice](#)
 NRICH: [Strike it Out](#)

Videos: www.tes.co.uk/teaching-resource/Number-bonds-to-ten-6413321

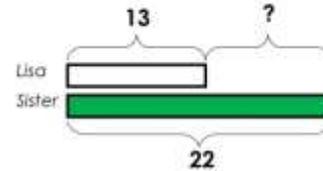
<p>Year 2</p>	<p>Find the difference</p>	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>	 <p>Count on to find the difference.</p>	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
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Use basic bar models with items to find the difference



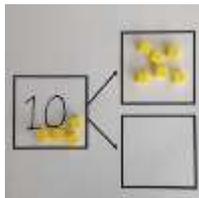
Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



Draw bars to find the difference between 2 numbers.

Part Whole Model

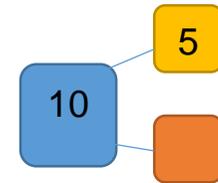
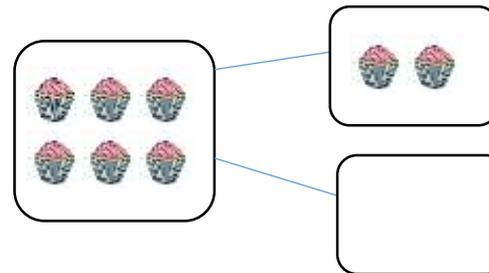


Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

$$10 - 6 =$$

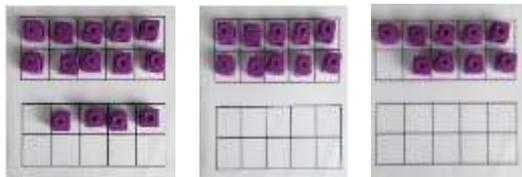
Use a pictorial representation of objects to show the part whole model.



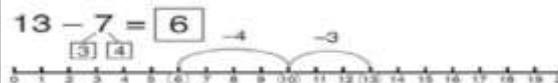
Move to using numbers within the part whole model.

Make 10

$$14 - 5 =$$



Make 14 on the ten frame. Take away the 4 first to make 10 and then takeaway 1 more so you have taken away 5. You are left with the answer of 9.



Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.

$$16 - 8 =$$

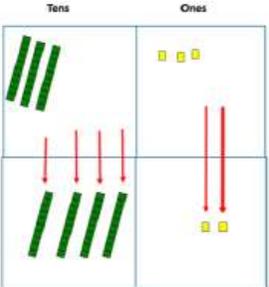
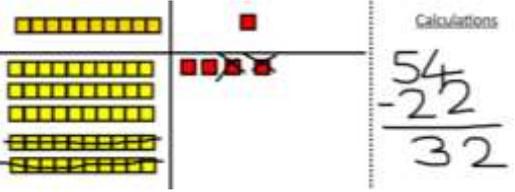
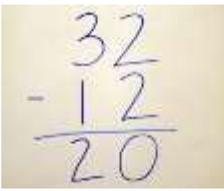
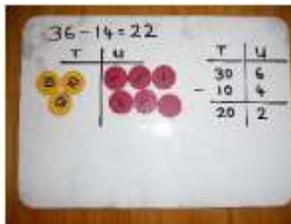
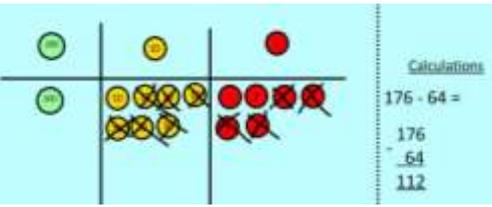
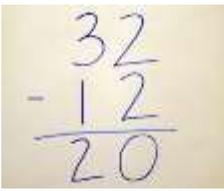
How many do we take off to reach the next 10?

How many do we have left to take off?

Additional Resources

- NRICH: [Butterfly Flowers](#)
- NRICH: [Number Round Up](#)
- NRICH: [4 Dom](#)

Videos: www.ncetm.org.uk/resources/40533

<p>Year 2 and Year 3</p>	<p>Column method without regrouping</p>	 <p>Use Base 10 to make the bigger number then take the smaller number away.</p>	<p>Draw the Base 10 or place value counters alongside the written calculation to help to show working.</p> 	<p>$47 - 24 = 23$</p> $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>This will lead to a clear written column subtraction.</p> 
<p>Show how you partition numbers to subtract. Again make the larger number first.</p>				

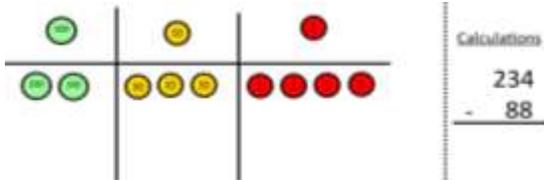
Additional Resources

- NRICH: [Super Shapes](#)
- NRICH: [Roll These Dice](#)
- NRICH: [Strike it Out](#)

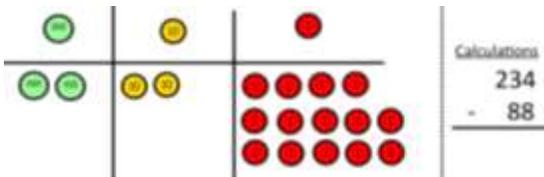
Videos: www.ncetm.org.uk/resources/40532 Two videos about subtraction

<p>Year 3 to Year 6</p>	<p>Column method with regrouping</p>	<p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p>	<p>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.</p>	
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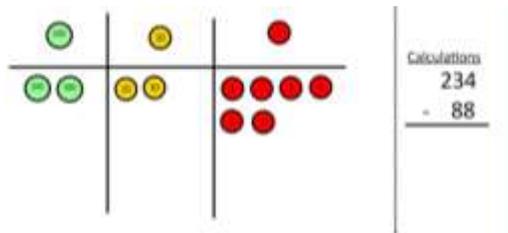
Make the larger number with the place value counters



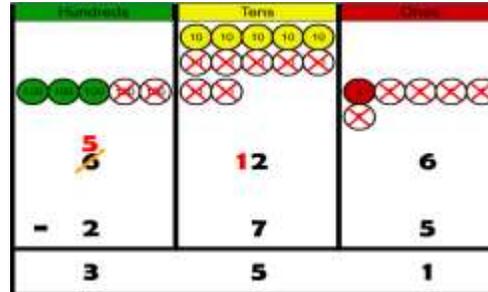
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Now I can subtract my ones.



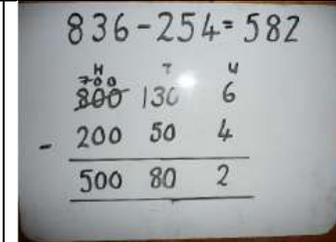
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



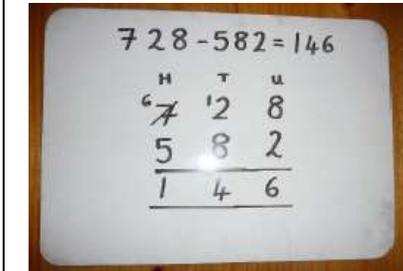
When confident, children can find their own way to record the exchange/regrouping.



Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



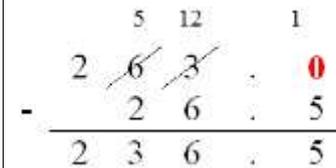
Children can start their formal

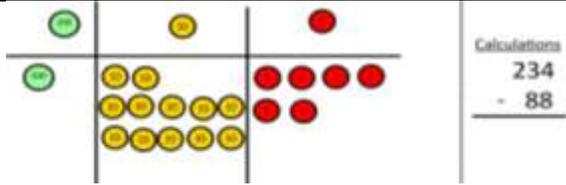


written method by partitioning the number into clear place value columns.

Moving forward the children use a more compact method.

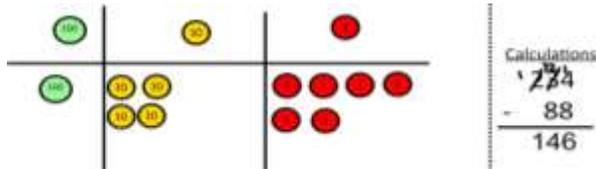
This will lead to an understanding of subtracting any number including decimals.





$$\begin{array}{r} \text{Calculations} \\ 234 \\ - 88 \\ \hline \end{array}$$

Now I can take away eight tens and complete my subtraction



$$\begin{array}{r} \text{Calculations} \\ \cancel{2}34 \\ - 88 \\ \hline 146 \end{array}$$

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Additional Resources

- NRICH: [Buying a Balloon](#)
- NRICH: [Take Three Numbers](#)
- NRICH: [Three Neighbours](#)
- NRICH: [Fifteen Cards](#)
- NRICH: [Up and Down Staircases](#)
- NRICH: [Magic Vs](#)
- NRICH: [Number Differences](#)
- NRICH: [Exploring Wild and Wonderful Number Patterns](#)
- NRICH: [Exploring Number Patterns You Make](#)
- NRICH: [Make 37](#)
- NRICH: [GOT IT](#)

Videos: www.ncetm.org.uk/resources/40532 three videos about subtraction

Multiplication

Key Vocabulary:

EYFS: groups of, lots of, times, altogether, multiply, count

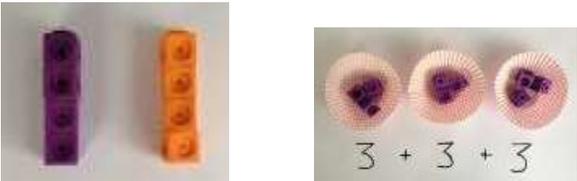
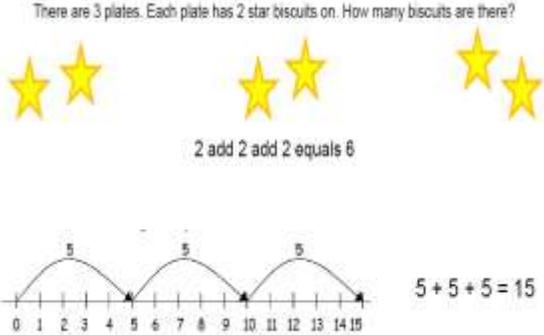
Year 1: As above plus: **array**

Year 2: As above plus: **multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...**

Year 3: As above plus: **partition, grid method, multiple, product, tens, ones, value.**

Year 4: As above plus: **total, inverse, square, factor, integer, decimal, ladder method, "carry".**

Year 5 and 6: As above plus: **compact multiplication.**

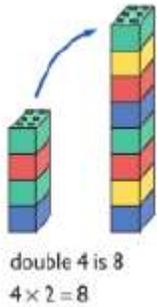
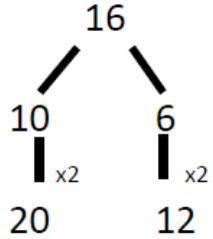
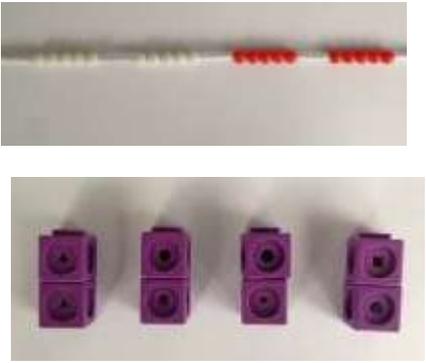
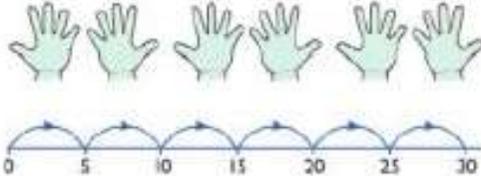
Taught at	Objective and Strategies	Concrete	Pictorial	Abstract
EYFS and Year 1	Repeated addition	 <p>Use different objects to add equal groups.</p>	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p> 	<p>Write addition sentences to describe objects and pictures.</p> 

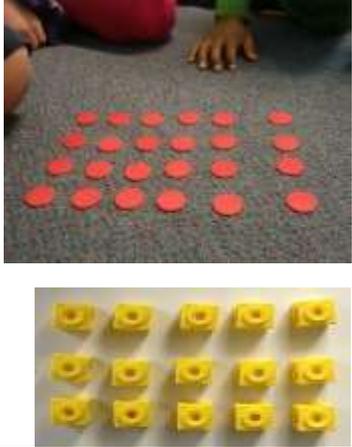
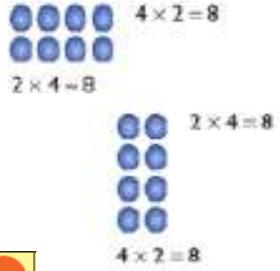
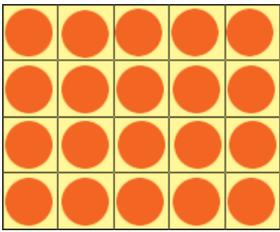
Additional Resources

NRICH EYFS: Maths Story Time

NRICH: Are You Well Balanced?

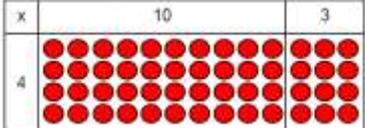
NRICH: Buzzy Bee

<p>Year 1 to year 3</p>	<p>Doubling</p>	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
<p>EYFS to year 4</p>	<p>Counting in multiples</p>	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

<p>Years 2 to 3</p>	<p>Arrays- showing commutative multiplication</p>	<p>Create arrays using counters/ cubes to show multiplication sentences.</p> 	<p>Draw arrays in different rotations to find commutative multiplication sentences.</p>   <p>Link arrays to area of rectangles.</p>	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p> $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$ </p>
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Additional Resources
 NRICH: [Clapping Times](#)
 NRICH: [Lots of Lollies](#)
 NRICH: [Growing Garlic](#)
 NRICH: [Ordering Cards](#)

Videos: www.ncetm.org.uk/resources/40530

<p>Year 3 to Year 5</p>	<p>Grid Method</p>	<p>Show the link with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move on to using Base 10 to move towards a more compact method.</p> <p>4 rows of 13</p>	<p>Children can represent the work they have done with place value counters in a way that they understand.</p>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1646 1077 1982 1173"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p>	x	30	5	7	210	35
x	30	5								
7	210	35								

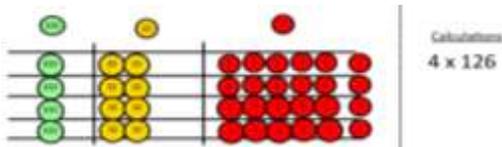
x	T	U
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	■ ■ ■ ■	■ ■
	■ ■ ■ ■	■ ■
	■ ■ ■ ■	■ ■

Move on to place value counters to show how we are finding groups of a number. We are

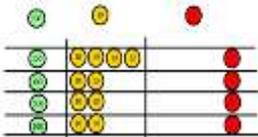
multiplying by 4 so we need 4 rows.



Fill each row with 126.



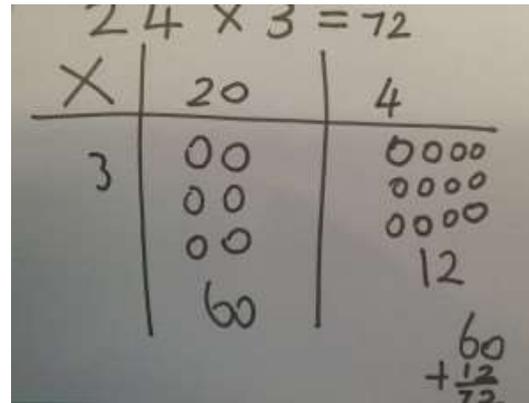
Add up each column, starting with the ones, making any exchanges needed.



Then you have your answer.



They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



	10	8
10	100	80
3	30	24

Progress to multiplying a 4-digit number by a two-digit number using grid method.

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

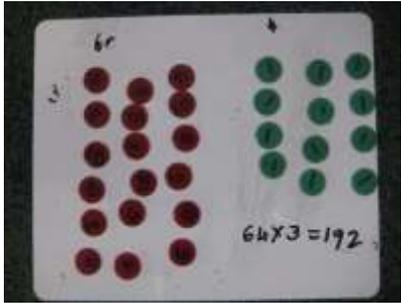
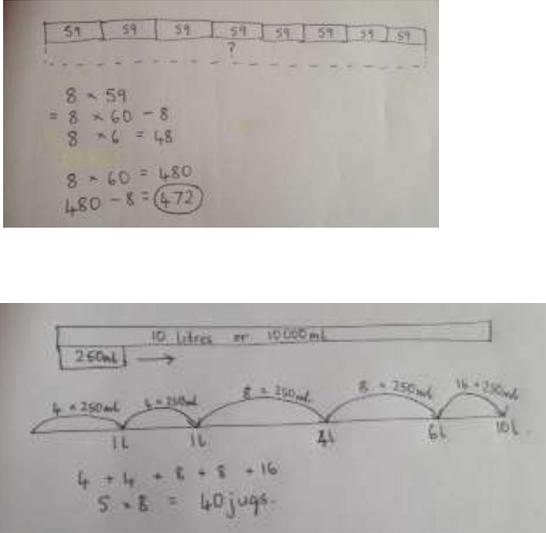
Additional Resources

NRICH: [Multiplication Square Jigsaw](#)

NRICH: [Shape Times Shape](#)

NRICH: [What do you need?](#)

NRICH: [Multiples Grid](#)

<p>Year 5 and Year 6</p>	<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication.</p>  <p>It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.</p>	<p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p> 	<p>Start with long multiplication (ladder method) reminding the children about lining up their numbers clearly in columns.</p> <p>If it helps, children can write out what they are solving next to their answer.</p> $ \begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array} $ <p>This moves to the more compact method.</p> $ \begin{array}{r} 53 \\ \times 24 \\ \hline 212 \\ 1060 \\ \hline 1272 \end{array} $
<p>Additional Resources NRICH: Music to my ears NRICH: Multiplication Squares NRICH: Flashing Lights NRICH: One Wasn't Square NRICH: Cycling Squares</p> <p>Videos: www.ncetm.org.uk/resources/40530 Moving from grid to column method www.ncetm.org.uk/resources/40533</p>				

Division

Key Vocabulary:

EYFS: share, count, group, set, double, half, share out.

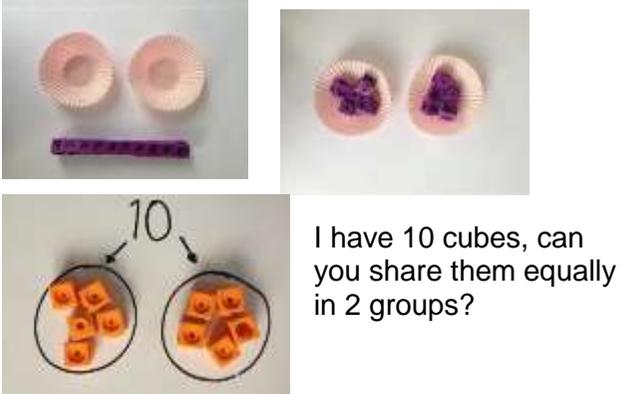
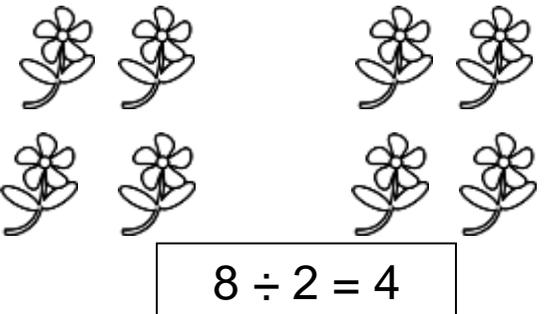
Year 1: As above plus: share equally, one each, two each, etc..

Year 2: As above plus: equal groups of, array, divide, divided into, division, grouping, left, left over.

Year 3: As above plus: inverse, remainder, multiple.

Year 4: As above plus: divisible by, factor.

Year 5 and 6: As above plus: bus stop method, quotient, divisor, dividend, prime number, prime factors, composite numbers, square root.

Taught at	Objective and Strategies	Concrete	Pictorial	Abstract
EYFS and Year 1	Sharing objects into groups	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>$8 \div 2 = 4$</p>	<p>Share 9 buns between three people.</p> <p>$9 \div 3 = 3$</p>

Additional Resources

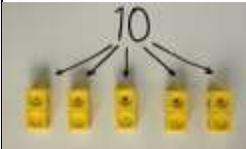
NRICH EYFS: Maths Story Time

NRICH: Lots of Biscuits

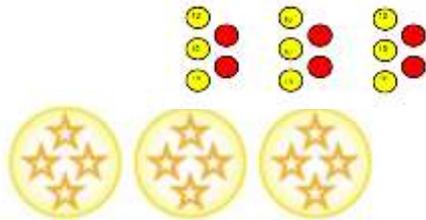
<http://www.taw.org.uk/lic/itp/grouping.html>

Year
2 to
Year
4

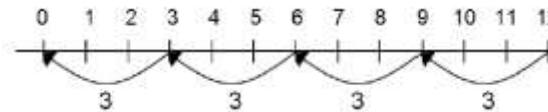
**Division
as
grouping**



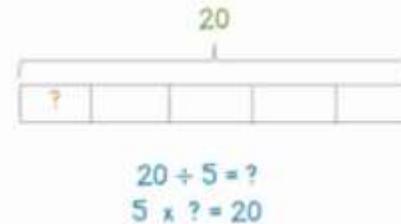
Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.



Use a number line to show jumps in groups. The number of jumps equals the number of groups.



Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?

Additional Resources

NRICH: Share Bears

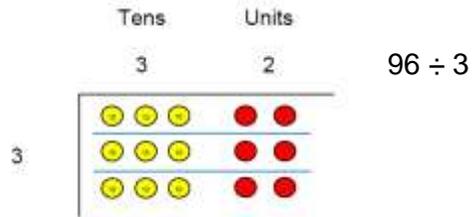
NRICH: Secret Number

NRICH: Journeys in Numberland

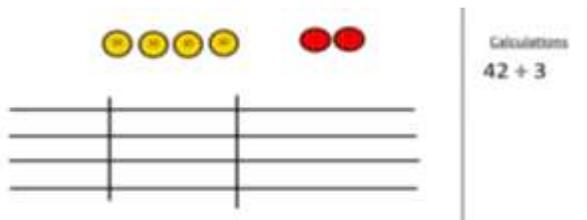
Videos: www.ncetm.org.uk/resources/43589

Short division

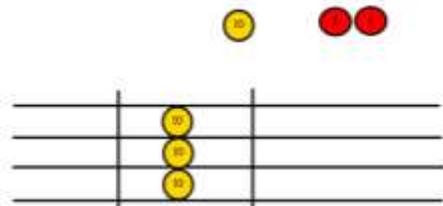
Use Place Value Counters



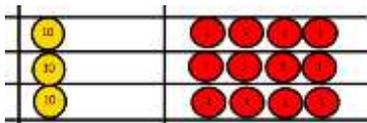
Progress to using place value counters to divide using the bus stop method alongside



Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

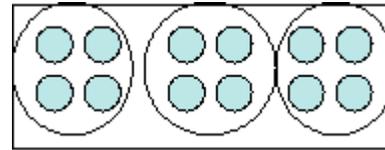


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \underline{8} \\ 7 \\ \underline{8} \\ 2 \\ \underline{2} \\ 0 \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{4} \\ 3 \\ \underline{3} \\ 2 \\ \underline{2} \\ 0 \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \\ 16 \\ \underline{16} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

Additional Resources

NRICH: [Flashing Lights](#)

NRICH: [Mystery Matrix](#)

NRICH: [Factor Lines](#)

NRICH: [Zios and Zepts](#)

NRICH: [Andy's Marbles](#)

NRICH: [Would you rather?](#)

NRICH: [Forgot the Numbers](#)

Videos: www.ncetm.org.uk/resources/43589